A special case of sinusoidal graphs over integers.

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This work is intended for educational purposes only.

Glossary of terms :

A sinusoid is a wave shaped like the graph of y = sin x but not necessarily odd.

$\lfloor n \rfloor$ is the floor function of n.

n is any number

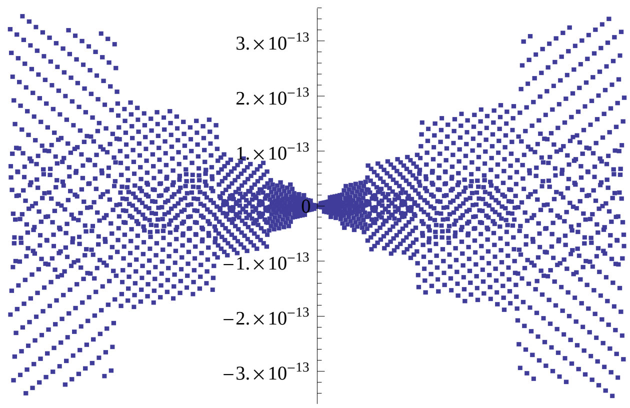
Abstract: After looking at the nature of a special case of a graph over integers, we observe the periods obtained by using {7, 9 and 10} digit precision of a certain constant. Finally we peek ahead at an extension of a family of graphs.

0. Introduction

What can we say about sin (⌊n⌋ * Pi * €) where € is an arbitrary constant? In the following pages we will concentrate on a special case where the function is sin (876 799 * ⌊n⌋ * Pi * e) and e = € / 876 799.
We are probably unaware of anything special about 876 799. That value will be critical only when we choose certain e's (m1, m2 and m3).

Lets graph a dashed line along the the n – axis with the graph of sin (⌊n⌋ * Pi)

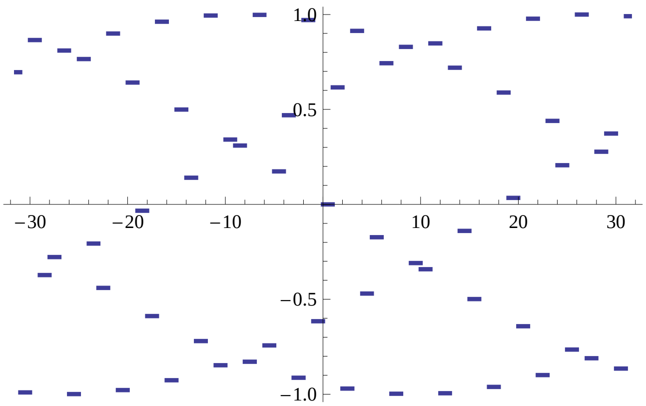
```
Plot[Sin[Floor[n] * Pi], {n, -1000, 1000},
 Axes → {False, True}, PlotStyle → Thick, PlotPoints → 400]
```

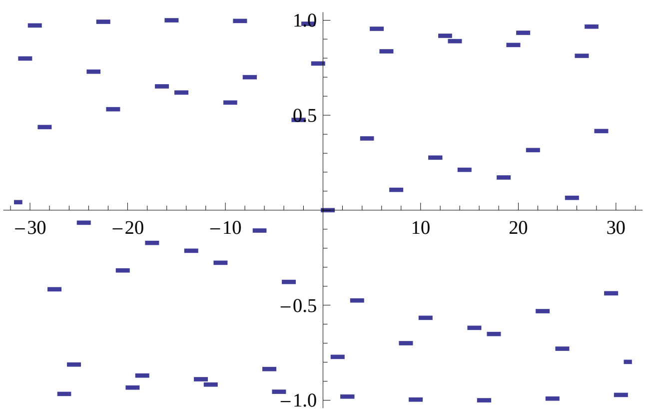


That didn't work out too well. We have now found the level of error tolerance in our grapher and the symmetry of the error of this grapher is remarkable. We will keep this level in mind but will make use of the symmetry of error toward the end of this paper.

Change sin (⌊n⌋*Pi)to sin (⌊n⌋*Pi*e)where e is a transcendental number. Since e represents the base of the natural log, we will let exp(1) be e and next we will try another transendental number 2^sqrt(2).

```
Plot[Sin[876 799 * Floor[n] * Pi * Exp[1]], {n, -10 * Pi, 10 * Pi}, PlotStyle → Thick]
Plot[Sin[876 799 * Floor[n] * Pi * 2^Sqrt[2]], {n, -10 * Pi, 10 * Pi}, PlotStyle → Thick]
```

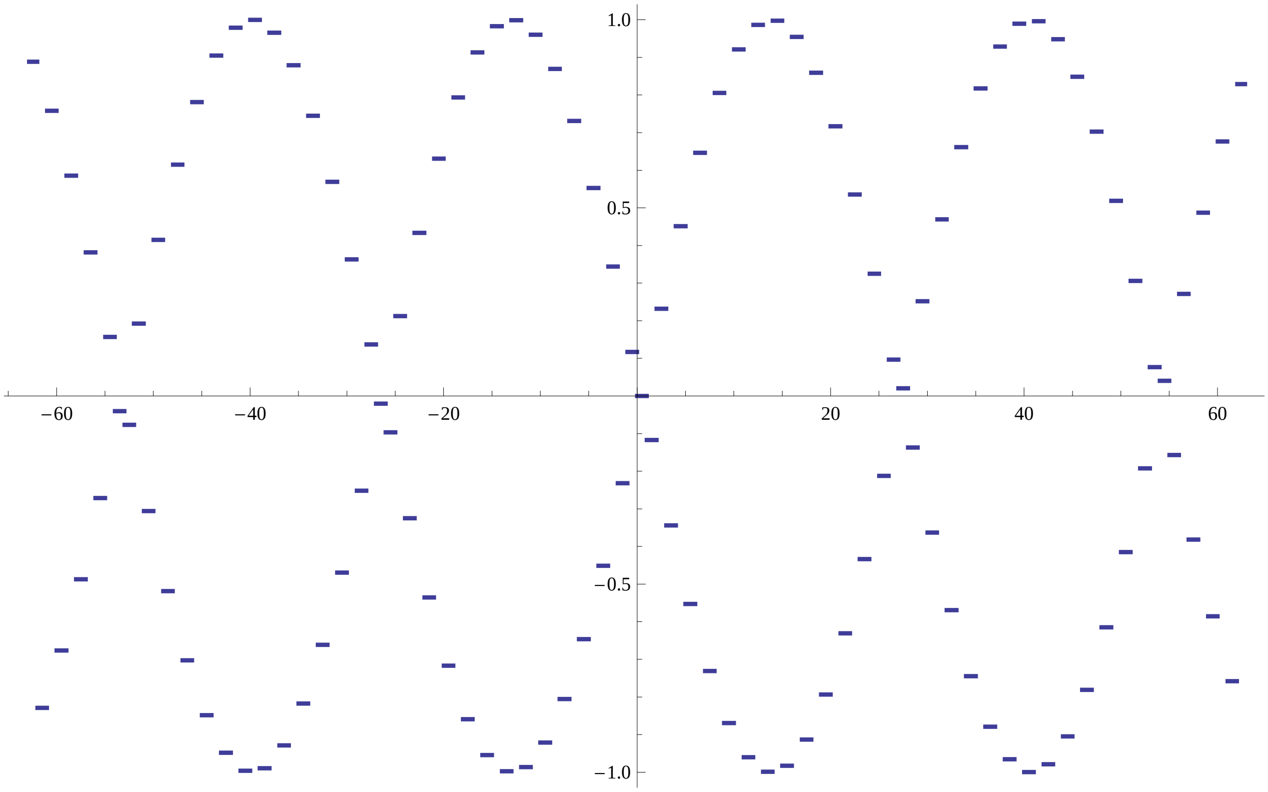




We are left with a graphs of complicated shapes.

Let' s change the value of our e to an irrational -- non transcendental -- number by using a sqrt(2) in our function.

```
Plot[Sin[876 799 * Floor[n] * Pi * Sqrt[2]], {n, -20 * Pi, 20 * Pi}, PlotStyle → Thick]
```



Now we have a graph that is a double sinusoid. In our few samples transcendental numbers produced complicated shapes and a non transcendental number produced a sinusoidal shape. This test does not work for all such numbers.

1. Candidate

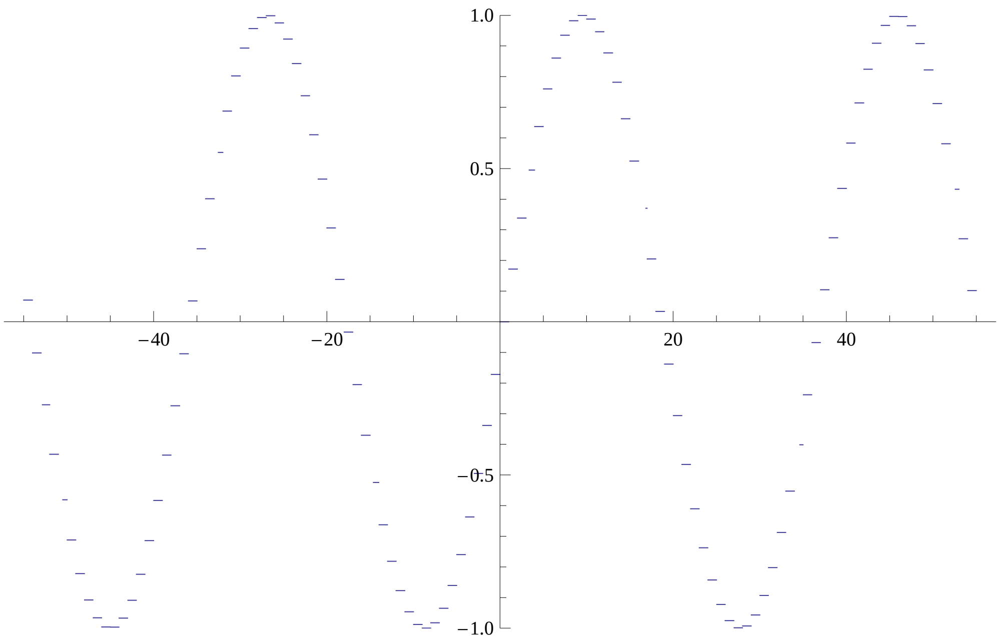
Consider the following graph with one such e that produces a sinusoidal graph.

The value we are going to use is 1/m1. m1 is an approximation of the MRB constant. 0

```
m1 = 1 878 596 / 10^7
N[%]
Plot[Sin[Pi * 876 799 * Floor[n] / m1],
 {n, -55, 55}, PlotRange → {-1, 1}, Axes → {True, True}]
```

$\frac{469\,649}{2\,500\,000}$

0.18786

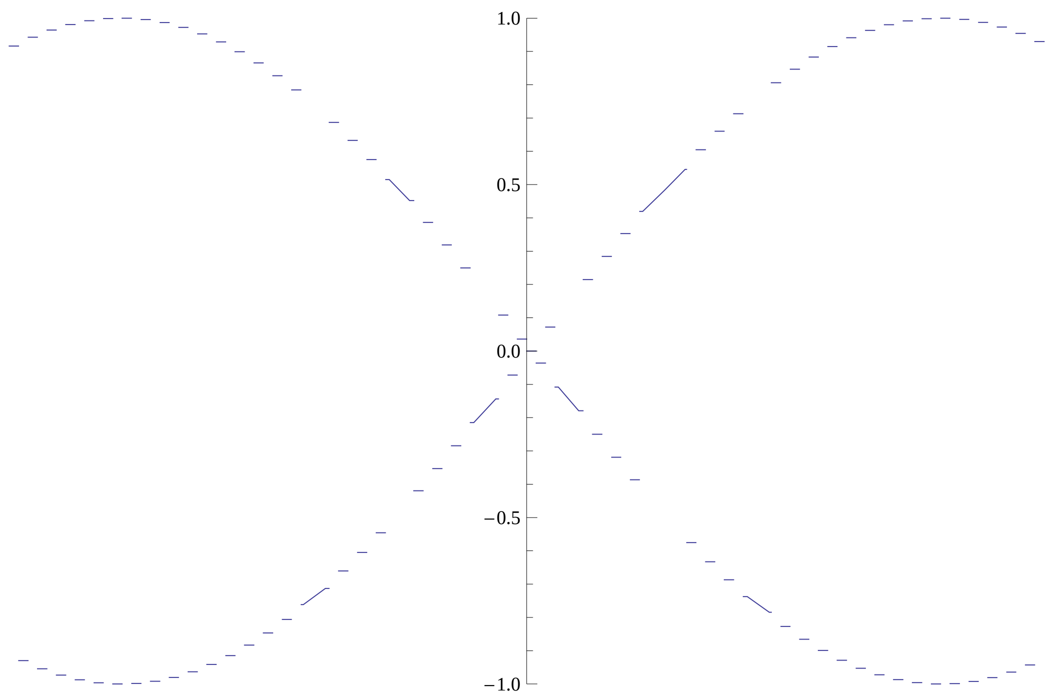


Since there are more digits to that constant, Lets use two more.

```
m2 = 187 859 642 / 10^9
N[%]
Plot[Sin[Pi * 876 799 * Floor[n] / m2], {n, -55, 55},
 PlotRange → {-1, 1}, Axes → {False, True}]
```

$$\frac{93\,929\,821}{500\,000\,000}$$

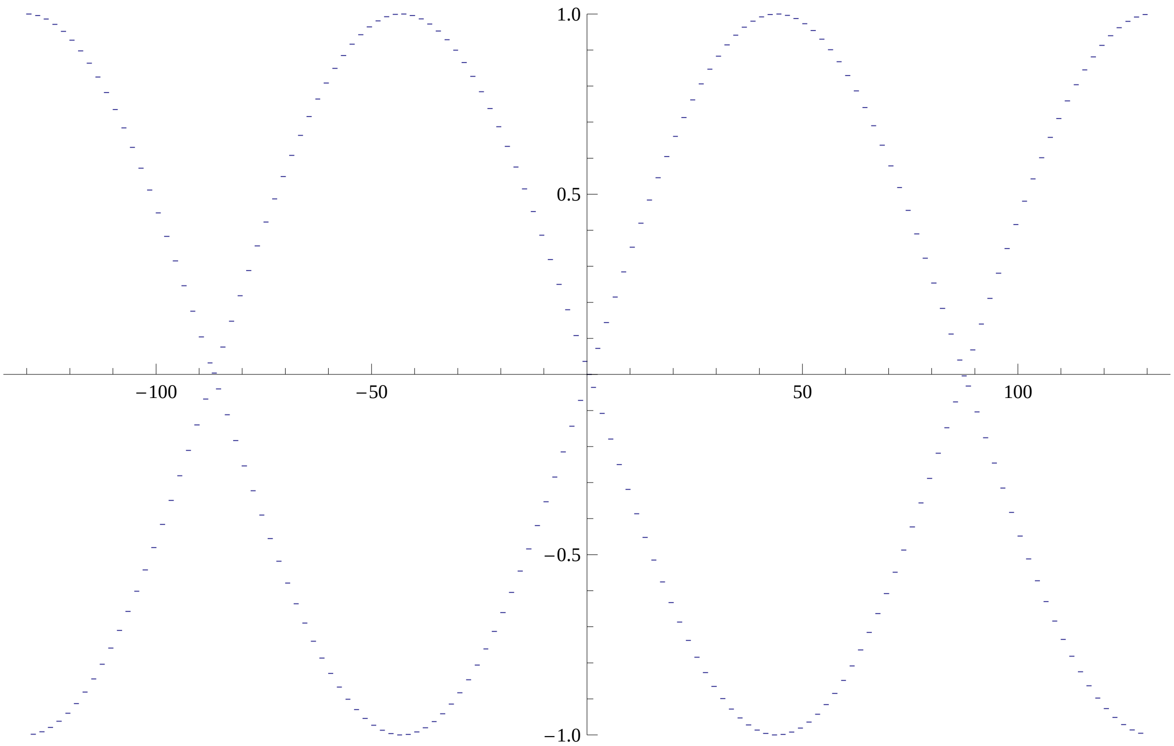
0.18786



That graph still could be sinusoidal.

To be sure that is sinusoidal lets look at it from a larger range.

```
Plot[Sin[Pi * 876 799 * Floor[n] / m2], {n, -130, 130},
 PlotRange → {-1, 1}, Axes → {True, True}]
```



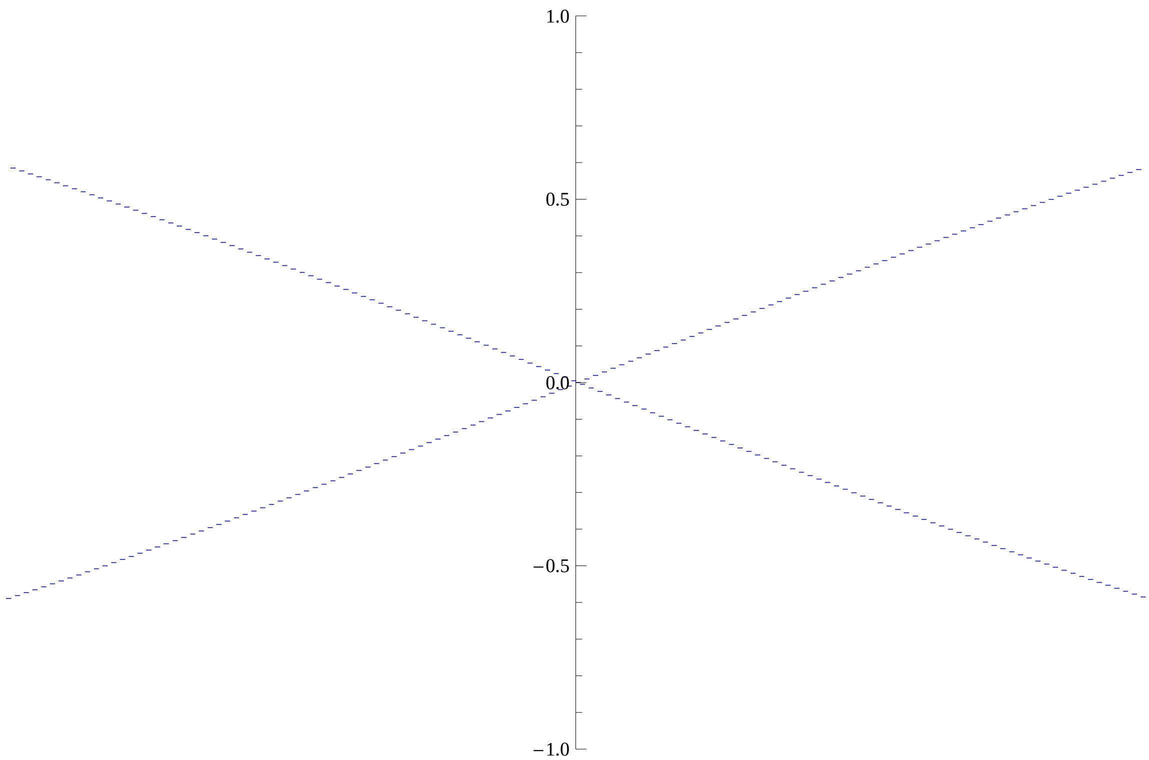
That's more like it.

If we want to see what happens when we add even 1 more digit to the approximation we had better use that new range instead of the old one.

```
m3 = 1 878 596 424 / 10^10
N[%]
Plot[Sin[Pi * 876 799 * Floor[n] / m3], {n, -130, 130},
 PlotRange → {-1, 1}, Axes → {False, True}]
```

$$\frac{234\,824\,553}{1\,250\,000\,000}$$

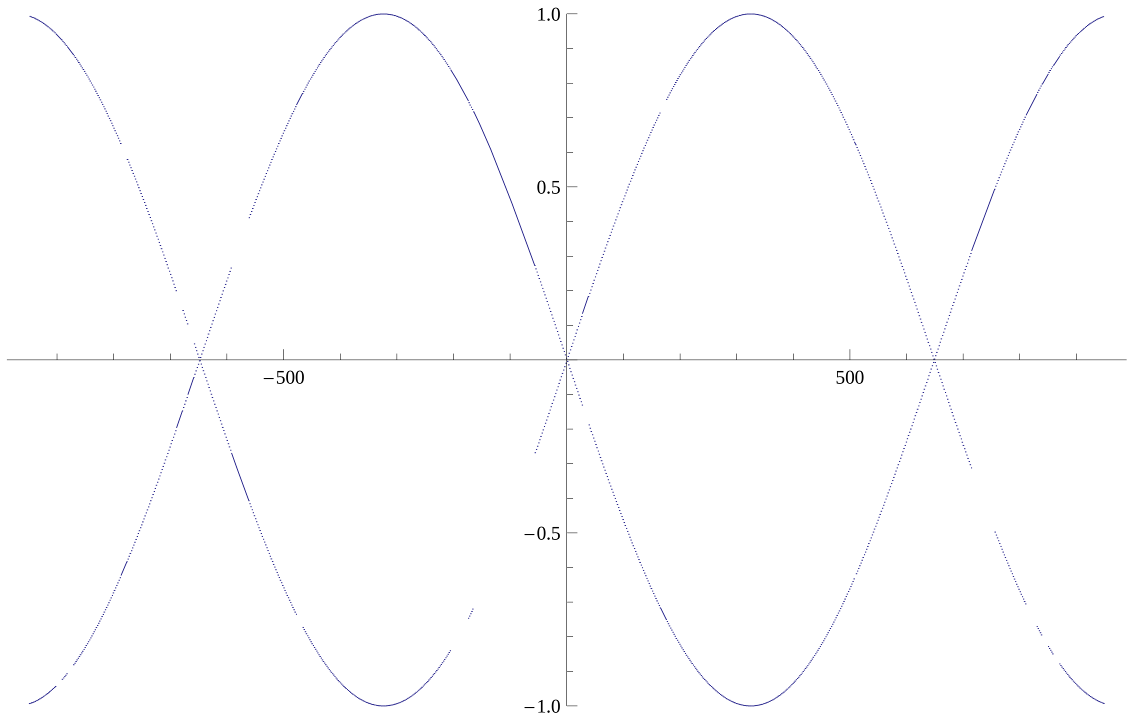
0.18786



What happened here? Did the graph suddenly decide not to be sinusoidal any more? Did we stumble upon some bifurcation?

To try to answer these questions we had better increase the range of the graph.

```
Plot[Sin[Pi*876799*Floor[n]/m3], {n, -949, 949},
 PlotRange → {-1, 1}, Axes → {True, True}]
```



That's better; it is sinusoidal after all. But now we wonder what is going on with that range? Why should we keep increasing it so? We know we are graphing a discontinuous function but we would have intuitively expected it to have approximately the same period for m1, m2 and m3.

2. Probe

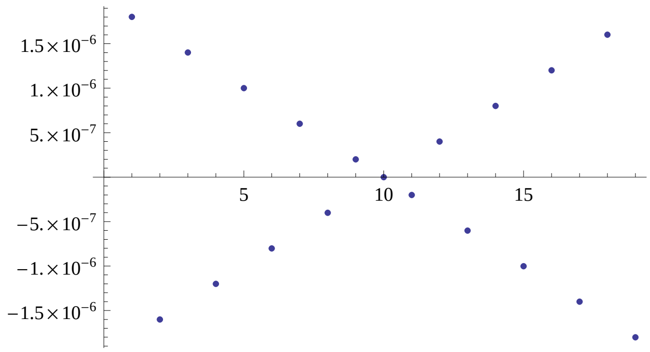
The following computation shows minimum values of the extended set of graphs. This will give us some hint as to the maximum size of the periods.

```
m1c = 1 878 596 424 620 671 202 485 / 10^22
N[%]
N[Table[Sin[Pi * 876 799 * Floor[n] / m1c], {n, -9, 9}]]
ListPlot[%]
```

$$\frac{375\,719\,284\,924\,134\,240\,497}{2\,000\,000\,000\,000\,000\,000\,000}$$

0.18786

$\{1.80174\times10^{-6}, -1.60154\times10^{-6}, 1.40135\times10^{-6}, -1.20116\times10^{-6}, 1.00097\times10^{-6}, -8.00772\times10^{-7}, 6.00579\times10^{-7}, -4.00386\times10^{-7}, 2.00193\times10^{-7}, 0., -2.00193\times10^{-7}, 4.00386\times10^{-7}, -6.00579\times10^{-7}, 8.00772\times10^{-7}, -1.00097\times10^{-6}, 1.20116\times10^{-6}, -1.40135\times10^{-6}, 1.60154\times10^{-6}, -1.80174\times10^{-6}\}$



Open Problem 0

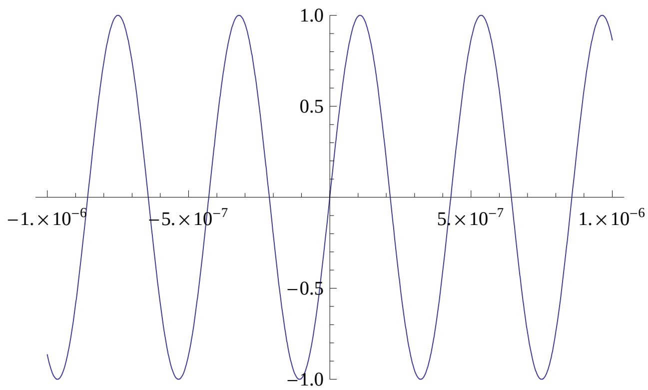
What is the precise maximum size of the periods?

Go back to §1. Take the expression we graphed and re-graph it without the floor function.

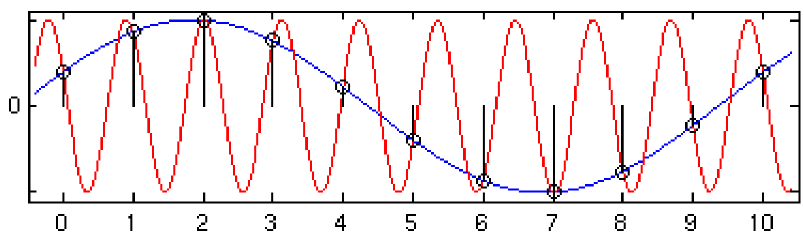
```
m1 = 1 878 596 / 10^7;
```

As expected we have a very small period of 2m1/876799

```
Plot[Sin[Pi * 876 799 * n / m1], {n, -.000001, .000001}, PlotRange → {-1, 1}, Axes → {True, True}]
2 m1 / 876 799.
```



4.28512 × 10^-7



In the above diagram we have the graph without the floor function overlaid by the graph with the floor function

Open Problem 1

Why do the graphs with the floor function intersect the graph without it in such a way to produce the previous sinusoids?

One last look

Combining the symmetry of error from the introduction to the

graphs from the candidate section gives the following graphs of interest.

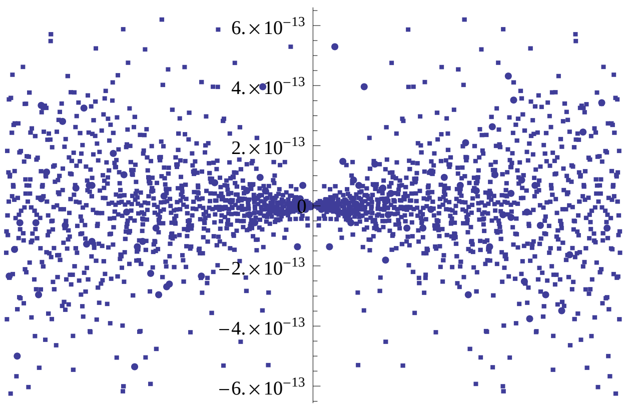
```
m1c = 1 878 596 424 620 671 202 485 / 10^22
```

```
m1 = 1 878 596 / 10^7
```

$\frac{469\,649}{2\,500\,000}$

$\frac{375\,719\,284\,924\,134\,240\,497}{2\,000\,000\,000\,000\,000\,000\,000}$

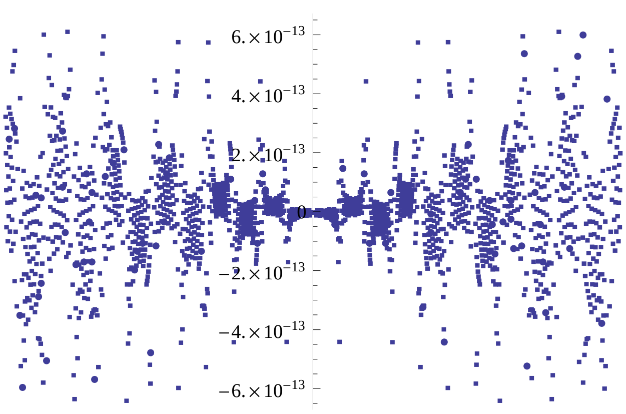
```
Plot[Sin[Floor[n] * Pi] / (Sin[Pi * 876 799 * Floor[n] / m1]),
 {n, -1000, 1000}, Axes → {False, True}, PlotStyle → Thick]
```




```
m2 = 187 859 642 / 10^9
```

$\frac{93\,929\,821}{500\,000\,000}$

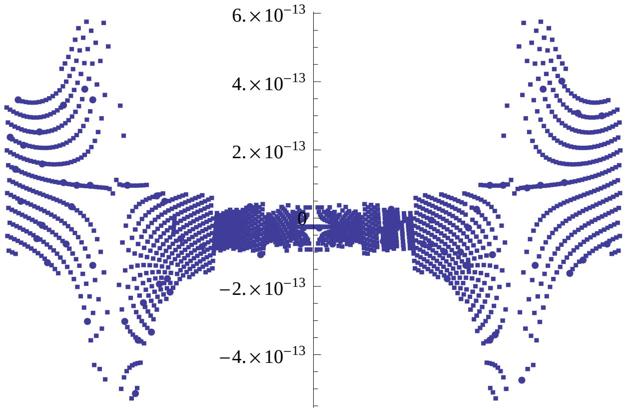
```
Plot[Sin[Floor[n] * Pi] / (Sin[Pi * 876 799 * Floor[n] / m2]),
 {n, -1000, 1000}, Axes → {False, True}, PlotStyle → Thick]
```



```
m3 = 1 878 596 424 / 10^10
```

$$\frac{234\,824\,553}{1\,250\,000\,000}$$

```
Plot[Sin[Floor[n] * Pi] / (Sin[Pi * 876 799 * Floor[n] / m3]),
 {n, -1000, 1000}, Axes → {False, True}, PlotStyle → Thick]
```



Open Problem 2

What expression in n best represents the symmetric error of our grapher?

Acknowledgements

The graphs were made by Mathematica 6.02 - Used by permission.
The one diagram was taken from Wikipedia and is in the public domain.

References

[0]
Accompanying paper called This is the MRB Constant
and
S.R.Finch, Mathematical Constants, Cambridge, 2003, pp.448 – 452.